



FERMILAB-Pub-85/30-T
February, 1985

Trying to Zero the
Cosmological Constant
with Conformal Symmetry

ROBERT D. PISARSKI
Fermi National Accelerator Laboratory
P.O. Box 500 Batavia, Illinois 60510

ABSTRACT

The spontaneous breaking of an exact conformal symmetry provides a mechanism for the zeroing of the cosmological constant, but it does so in a manner which violates either unitarity or known phenomenology.

There are a few reasons known to explain why the cosmological constant should be as small as it is observed to be. In this note, I show how the spontaneous breaking of conformal symmetry, in a unified theory with exact conformal symmetry, can produce zero cosmological constant without requiring other dimensional parameters, such as Newton's constant, masses, and so on, to vanish. Unfortunately, it appears as if the "conformal" solution to the cosmological constant problem is like that of supersymmetry¹: it works, but it does not apply to the universe we find ourselves in. For supersymmetry, this is simply because our universe is not supersymmetric; it is less obvious why the conformal solution does not apply. I present my arguments in the hope I might have overlooked some loophole.

The suspicion that the cosmological constant problem could be related to spontaneously broken conformal symmetry is based on an analogy which, while suggestive, is not precise. Suppose one has a unified theory, of both gravity and matter, that is conformally symmetric. This means that the effective action must be invariant under local conformal transformations generated by $\lambda(x)$:

$$\begin{aligned} & A \left(g_{\mu\nu}(x), \phi(x), \psi(x), A_\mu(x) \dots \right) \\ &= A \left(e^{2\lambda(x)} g_{\mu\nu}(x), e^{-\lambda(x)} \phi(x), e^{-3/2\lambda(x)} \psi(x), A_\mu(x) \dots \right) . \end{aligned} \tag{1}$$

The effective action A is a functional over background fields from the metric, $g_{\mu\nu}(x)$ (and, possibly, spin- $\frac{3}{2}$ fields if there is local supersymmetry), and matter; $\phi(x)$, $\psi(x)$, and $A_\mu(x)$ represent any number of spin-0, $-\frac{1}{2}$, and -1 fields, respectively.

By Noether's theorem, corresponding to global conformal transformations, $\lambda(x) = \text{constant}$, there is a dilatation current, J_D^μ . For any theory, the trace of the stress-energy tensor is equal to the divergence of the dilatation current, $T^\mu_\mu = J_{D,\mu}^\mu$. If the theory, and its vacuum, are conformally symmetric, we see that $T^\mu_\mu = J_{D,\mu}^\mu = 0$: the explicit symmetry only allows dimensionless couplings in the action.

Suppose, however, that while the theory is conformally symmetric, the vacuum that the theory chooses is not. Suppose further that this conformally asymmetric vacuum has flat space-time. The stress-energy tensor, as I am using it here, is complete, and includes the contribution of all matter and gravitational fields. Thus the vacuum expectation value of its trace is directly related to the cosmological constant, $\Lambda \sim \langle T^\mu_\mu \rangle$. As for the dilatation current, it will act in a non-trivial fashion on the vacuum, but the remnant of the exact symmetry in the original theory is that matrix elements of its divergence vanish at zero momentum, as $(\text{momentum})^2$. Consequently, the conformal symmetry requires the cosmological constant to vanish: $\Lambda \sim \langle T^\mu_\mu \rangle = \langle J_{D,\mu}^\mu \rangle = 0$.

This argument is not as trivial as it might first appear. If one had a theory which was scale symmetric in flat but not curved space-time², a scale asymmetric vacuum might have $\Lambda = 0$ classically, but generally a non-zero value for Λ will be generated by loop effects. This, of course, is the usual problem with the cosmological constant. In contrast, for a theory which is conformally symmetric in curved space-time, if it can be shown that the vacuum is scale asymmetric and has flat space-time, not only is $\Lambda = 0$ classically, but the conformal symmetry ensures that it will stay zero, even when the effects of any interactions, perturbative or not, are included. In other words, in

most theories, while a vacuum with flat space-time will be stable with respect to fluctuations in the matter fields, this does not imply that the vacuum is stable to leading order in $g_{\mu\nu}$; if $\Lambda \neq 0$ is produced by quantum effects, the vacuum is not. With conformal symmetry, however, if the vacuum has flat space-time to begin with, the conformal symmetry guarantees that it will always remain stable to leading order in $g_{\mu\nu}$, $\Lambda = 0$. This does not mean that in conformally symmetric theories, that any asymmetric vacuum need have flat space-time--this is determined by the dynamics--and only stability to leading order in $g_{\mu\nu}$ is assured. Even so, this is far more than what one would have without conformal symmetry, since the asymmetric vacuum may have non-zero values for masses and other dimensional parameters, with the sole exception of Λ .

In the following, I demonstrate the correctness of this naive intuition. For simplicity, I take the symmetry breaking to be due to one, and then several, scalar fields, including only the effects of these fields and of the metric in the analysis. Alternately, the symmetry breaking could be due to composite operators for higher spin fields, such as $\text{tr}(\bar{\Psi}\Psi)$ or $\text{tr}(F_{\mu\nu}^2)$, but this does not alter the conclusions.

On one point, the preceding argument is rather misleading. As a system with a spontaneously broken symmetry, shouldn't there be a massless, spin-zero excitation, a dilaton, generated by the action of the dilatation current on the vacuum? For the case of a single scalar field, with non-zero vacuum expectation value, the couplings of that scalar field do end up looking like those of a dilaton. When several scalar fields develop non-zero vacuum expectation values though, usually the only massless field is the graviton, which manifestly is not

spin-zero.

Why is there no dilaton,⁴ even when the asymmetric vacuum has flat space-time? The reason lies in the form of the dilatation current. Take a single scalar field, ϕ , conformally coupled to the metric. The kinetic energy of the field, $\frac{1}{2}(\partial_\mu \phi)^2$, contributes $\phi \partial^\mu \phi$ to J_D^μ , while the conformal coupling, $-(1/12)R\phi^2$, contributes $-\phi \partial^\mu \phi$ to J_D^μ , with the total zero. The same thing happens for fermions- the contribution of the kinetic energy to J_D^μ cancels against that of the spin connection. Since spin-1 fields are conformally invariant, eq. (1), we see that matter fields do not contribute at all to J_D^μ .

It must be emphasized that the assumption of exact conformal symmetry greatly restricts the class of theories to which my arguments apply. For the coupling of the metric tensor to itself to be conformally symmetric, only a term $\sim C_{\alpha\beta\gamma\delta} C^{\alpha\beta\gamma\delta}$ ($C_{\alpha\beta\gamma\delta}$ = Weyl tensor) is permitted, to the exclusion of an Einstein term, etc. Hence the purely gravitational part of the action must involve higher derivatives. While there is some hope that such theories are unitary in spite of their higher derivatives,^{5,6} this is far from certain.

Even for a theory which is conformally symmetric at the classical level, usually quantum effects will destroy this symmetry through the conformal anomaly⁷. To retain the exact conformal symmetry, I need to insist that the unified theory be finite⁸. That is, for each and every coupling of matter and gravity, the corresponding β -function must vanish. It is possible that there are no theories which satisfy these conditions, although Fradkin and Tseytlin⁸ have advanced several possible candidates.

Going beyond the usual confines of field theory, Englert, Truffin, and Gastmans⁹ have proposed modifying theories in 4- ϵ dimensions so that they have no conformal anomaly but non-zero β -functions in four dimensions. Antoniadis and Tsamis⁶ have used their mechanism to relate conformal symmetry breaking to the cosmological constant in these theories. While our assumptions are very different, the arguments that I give are sufficiently general that they apply to the theories of Antoniadis and Tsamis as well.

With these qualifications aside, I turn to the analysis.

I. SINGLE SCALAR FIELD

Let the scalar field which acquires a vacuum expectation value be ϕ . I assume that the vacuum has flat space-time, $g_{\mu\nu} = \eta_{\mu\nu}$, with $\langle\phi\rangle = \phi_0$. Defining $\phi_q = \phi - \phi_0$, I expand the effective action to quartic order in ϕ_q , including all terms up to two derivatives:

$$\begin{aligned}
 A \approx \int d^4x \sqrt{g} \left\{ -\frac{1}{16\pi G} (R - 2\Lambda) \right. \\
 - \frac{1}{2} Z [g^{\mu\nu} (\partial_\mu \phi_q) (\partial_\nu \phi_q) + m^2 \phi_q^2 + g_3 \phi_q^3 + g_4 \phi_q^4 \\
 \left. + c R \phi_q + d R \phi_q^2 \right\} .
 \end{aligned} \tag{2}$$

Z represents a (finite) term for wave-function renormalization of ϕ . A term linear in ϕ_q is excluded by the premise that the true vacuum has $\langle\phi\rangle = \phi_0$ in flat space-time. Under the assumptions given, there are other terms which could contribute to A , such as $g^{\mu\nu} (\partial_\mu \phi_q) (\partial_\nu \phi_q) \phi_q$, but by the arguments to follow these can be shown not to contribute.

Except for ϕ and $g_{\mu\nu}$, the dependence of A on all other fields in the problem is ignored.

The local conformal symmetry of the theory is not manifest in the effective action of eq. (2). It does imply relations between the various terms in eq. (2), which can be obtained merely by using the invariance of A under global conformal transformations. To first order in $\lambda(x) = \lambda$,

$$\frac{\delta A}{\delta \lambda} = \int d^4x \left\{ 2g_{\mu\nu}(x) \frac{\delta A}{\delta g_{\mu\nu}(x)} - \phi(x) \frac{\delta A}{\delta \phi(x)} \right\} = 0 \quad (3)$$

evaluated at any point in the function space of $g_{\mu\nu}(x)$ and $\phi(x)$. For the vacuum state, this gives

$$\begin{aligned} \int d^4x \frac{\Lambda}{8\pi G} &= \frac{1}{2} \int d^4x \left. \eta_{\mu\nu} \frac{\delta A}{\delta g_{\mu\nu}(x)} \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} \\ &= \frac{1}{2} \int d^4x \left. \phi_0 \frac{\delta A}{\delta \phi(x)} \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = 0 \quad . \end{aligned} \quad (4)$$

Thus the most elementary Ward identity of conformal symmetry, eq. (3), suffices to show that $\Lambda = 0$ in a conformally asymmetric vacuum with flat space-time.

I proceed now in a manner which, while not the most clever, is most directly generalizable to the case where several scalar fields develop vacuum expectation values.

To second order in λ ,

$$\left. \frac{\delta^2 A}{\delta \lambda^2} \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = \iint \left\{ 4\eta_{\mu\nu}\eta_{\rho\sigma} \frac{\delta^2 A}{\delta g_{\mu\nu}\delta g_{\rho\sigma}} \right. \quad (5)$$

$$\left. -4\eta_{\mu\nu}\phi_0 \frac{\delta^2 A}{\delta g_{\mu\nu}\delta\phi} + \phi_0^2 \frac{\delta^2 A}{\delta\phi^2} \right\} \Bigg|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = 0 \quad .$$

For notational ease, the explicit dependence of the fields on the space-time coordinates x_μ has been dropped. Using the identity

$$\int \phi \frac{\delta}{\delta\phi} \left(\frac{\delta A}{\delta\lambda} \right) \Bigg|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} \quad (6)$$

$$= \iint \left\{ 2g_{\mu\nu}\phi \frac{\delta^2 A}{\delta g_{\mu\nu}\delta\phi} - \phi^2 \frac{\delta^2 A}{\delta\phi^2} \right\} \Bigg|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = 0 \quad ,$$

eq. (5) becomes

$$\iint 4\eta_{\mu\nu}\eta_{\rho\sigma} \frac{\delta^2 A}{\delta g_{\mu\nu}\delta g_{\rho\sigma}} \Bigg|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = \iint \phi_0^2 \frac{\delta^2 A}{\delta\phi^2} \Bigg|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} \quad (7)$$

To evaluate this, one defines $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and expands the Einstein action to quadratic order in $h_{\mu\nu}$. It is necessary to use the conformal gauge, $h_{\mu\nu}{}^{;\nu} = \frac{1}{2} h_{\nu}{}^{\nu}{}_{;\mu}$, so that gauge-fixing terms do not contribute. Then

$$\left. \iint \eta_{\mu\nu} \eta_{\rho\sigma} \frac{\delta^2 A}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = \iint + \frac{3}{16\pi G} (-\partial^2) \quad .$$

Since

$$\left. \iint \phi_0^2 \frac{\delta^2 A}{\delta \phi^2} \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = \iint -Z \phi_0^2 (-\partial^2 + m^2) \quad ,$$

Eq. (7) gives

$$\text{and } m^2 = 0, \tag{8}$$

$$\frac{1}{16\pi G} = -\frac{Z}{12} \phi_0^2 \quad . \tag{9}$$

The conformal symmetry requires ϕ_0 to be massless. Moreover, if Newton's constant is to have the right sign, $G > 0$, the scalar field must be a ghost-like field, $Z < 0$ (for real ϕ_0).

One continues in a similar vein, using various conformal Ward identities to obtain further restrictions on A. From

$$\begin{aligned}
 & \left. \iint \phi_0^2 \frac{\delta^2}{\delta \phi^2} \left(\frac{\delta A}{\delta \lambda} \right) \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} \\
 & = \iiint \left\{ 2\eta_{\mu\nu} \phi_0^2 \frac{\delta^3 A}{\delta g_{\mu\nu} \delta \phi^2} - 2\phi_0^2 \frac{\delta^2 A}{\delta \phi^2} \right. \quad (10) \\
 & \quad \left. - \phi_0^3 \frac{\delta^3 A}{\delta \phi^3} \right\} \Big|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = 0
 \end{aligned}$$

Only the kinetic energy of the ϕ_q field contributes to the first two terms on the right hand side of eq. (10), and its contribution cancels between them. This leaves

$$\begin{aligned}
 & \left. \iiint \phi_0^3 \frac{\delta^3 A}{\delta \phi^3} \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = 0 \quad , \quad (11)
 \end{aligned}$$

which requires the trilinear ϕ_q^3 coupling to vanish,

$$g_3 = 0 \quad , \quad (12)$$

and excludes a term as $g^{\mu\nu} (\partial_\mu \phi_q) (\partial_\nu \phi_q) \phi_q$. From

$$\left. \int \int \int \phi_0^3 \frac{\delta^3}{\delta \phi^3} \left(\frac{\delta A}{\delta \lambda} \right) \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = \int \int \int \int \left\{ 2\phi_0^3 g_{\mu\nu} \frac{\delta^4 A}{\delta g_{\mu\nu} \delta \phi^3} \right. \quad (13)$$

$$\left. - 3\phi_0^3 \frac{\delta^3 A}{\delta \phi^3} - \phi_0^4 \frac{\delta^4 A}{\delta \phi^4} \right\} \Big|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = 0 \quad ,$$

since there are no terms cubic in ϕ_q , this becomes

$$\left. \int \int \int \int \phi_0^4 \frac{\delta^4 A}{\delta \phi^4} \right|_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \phi = \phi_0}} = 0 \quad , \quad (14)$$

so the four-point ϕ_q coupling vanishes,

$$g_4 = 0 \quad , \quad (15)$$

and a term as $g^{\mu\nu} (\partial_\mu \phi_q) (\partial_\nu \phi_q) \phi_q^2$ is prohibited.

To fix the remaining terms in the effective action, I consider the conformal Ward identities at a point where $\phi = \phi_0$, but $g_{\mu\nu}$ is arbitrary. Eq. (3) gives rise to terms $\sim R$, which upon use of eq. (3), determines the $R\phi_q$ term:

$$c = -\frac{1}{3} \phi_0 \quad . \quad (16)$$

Likewise, evaluating eq. (6) at this point gives

$$d = -1/6 \quad . \quad (17)$$

We see that the two, three, and four point terms for ϕ_q vanish. It is easy to show that this is true for all couplings of ϕ_q at zero momentum. The effective potential for $\phi, V(\phi)$, is obtained from the effective action by requiring all fields to be constant in space-time:

$$A_{\text{constant}}^{g_{\mu\nu}, \phi} = -\frac{1}{2} \int d^4x \sqrt{g} V(\phi) \quad . \quad (18)$$

If we plug this into eq. (3), and set $g_{\mu\nu} = \eta_{\mu\nu}$, but leave ϕ arbitrary, we can solve for $V(\phi)$:

$$V(\phi) = g_4 \phi^4 \quad (19)$$

In theories with renormalization, non-trivial forms for $V(\phi)$ can develop at the quantum level, since then a renormalization mass scale μ enters, and functions of ϕ/μ can occur. In a finite, conformally symmetric theory, the bare and renormalized potentials must each have the same simple form, $\sim \phi^4$.

For $g_4 \neq 0$, the only vacuum is flat space-time is $\phi = 0$. When $g_4 = 0$, without a potential for ϕ , any value of $\phi = \phi_0$ will do.

Consequently, and while it was not obvious at first, the cosmological constant is zero because the ϕ self-coupling is set to zero, at least implicitly.¹⁰ In a finite theory, this is not entirely arbitrary, since if all β -functions vanish, the bare value of each coupling must be specified as initial data for our universe. More to the point, this will no longer be true when more than one scalar field develops a non-zero vacuum expectation value.

The effective action can be worked out as before. The result is

$$A \approx -Z \int d^4x \sqrt{g} \left\{ + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi_q) (\partial_\nu \phi_q) - \frac{1}{12} R (\phi_0 + \phi_q)^2 + g_4 (\phi_0 + \phi_q)^4 \right\} . \quad (20)$$

Newton's constant is given by eq. (9), and the cosmological constant can be read off from eq. (20) as

$$\frac{\Lambda}{8\pi G} = -Z g_4 \phi_0^4 , \quad (21)$$

For any value of g_4 , the conformally asymmetric vacuum is $\phi = \phi_0$, in a space-time with $R = 4\Lambda$. For $Z < 0$ and $g_4 > 0$, this corresponds to de Sitter space-time.

What about the fact that ϕ_q is a ghost-like field, $Z < 0$? Here an old trick of Deser's¹¹ can be used. Redefine the metric tensor as

$$g'_{\mu\nu} = \left(\frac{\phi_0 + \phi_q}{\phi_0} \right)^2 g_{\mu\nu} . \quad (22)$$

In terms of $g'_{\mu\nu}$, the effective action is

$$A \approx \int d^4x \sqrt{g'} \frac{-1}{16\pi G} (R' - 2\Lambda) , \quad (23)$$

where the Ricci scalar R' is constructed from $g'_{\mu\nu}$. Hence the ghost field has disappeared, and we are left with Einstein gravity. At least in this instance, the ghost field can be eliminated by the proper choice of a conformal gauge.

SEVERAL SCALAR FIELDS

The generalization to the case where several scalar fields have non-zero vacuum expectation values is immediate. Besides ϕ , I take the other scalar fields to be $\tilde{\phi}^i$, $\tilde{\phi}^i = \tilde{\phi}_0^i + \tilde{\phi}_q^i$. I assume that in flat space-time, $\langle \tilde{\phi}^i \rangle = \tilde{\phi}_0^i$. For the effective action,

$$\begin{aligned}
 A \approx & \int d^4x \sqrt{g} \left\{ -\frac{1}{16\pi G} (R-2\Lambda) - Z \left(\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi_q) (\partial_\nu \phi_q) \right. \right. \\
 & + \frac{1}{2} m^2 \phi_q^2 + g_3 \phi_q^3 + g_4 \phi_q^4 \left. \right) + \sum_i \frac{1}{2} g^{\mu\nu} (\partial_\mu \tilde{\phi}_q^i) (\partial_\nu \tilde{\phi}_q^i) \\
 & + \frac{1}{2} \tilde{m}_i^2 (\tilde{\phi}_q^i)^2 + \sum_{i,j,k} \tilde{g}_3^{ijk} \tilde{\phi}_q^i \tilde{\phi}_q^j \tilde{\phi}_q^k + \sum_{i,j,k,l} \tilde{g}_4^{ijkl} \tilde{\phi}_q^i \tilde{\phi}_q^j \tilde{\phi}_q^k \tilde{\phi}_q^l \left. \right\}
 \end{aligned} \quad (24)$$

This is not the most general form of the effective action possible, but it will serve to illustrate the essential points. The fields $\tilde{\phi}^i$ are all taken to be physical scalars, with wave function renormalization = +1. Their coupling, \tilde{g}_3^{ijk} and \tilde{g}_4^{ijkl} , are symmetric in all indices. Terms $\sim R\phi_q$, $\sim R\tilde{\phi}_q^i$, etc., are ignored. Eq. (3) becomes:

$$\frac{\delta A}{\delta \lambda} = \int \left\{ 2g_{\mu\nu} \frac{\delta A}{\delta g_{\mu\nu}} - \phi \frac{\delta A}{\delta \phi} - \sum_i \tilde{\phi}^i \frac{\delta A}{\delta \tilde{\phi}^i} \right\} = 0 \quad . \quad (25)$$

Since terms linear in ϕ and $\tilde{\phi}^i$ are excluded by assumption, eq. (25) evaluated at $g_{\mu\nu} = \eta_{\mu\nu}$, $\phi = \phi_0$, and $\tilde{\phi}^i = \tilde{\phi}_0^i$, gives

$$\Lambda = 0 \quad .$$

Eq. (7) becomes

$$\begin{aligned}
 & \left. \iint 4\eta_{\mu\nu}\eta_{\rho\sigma} \frac{\delta^2 A}{\delta g_{\mu\nu} \delta g_{\rho\sigma}} \right| \\
 & \quad g_{\mu\nu} = \eta_{\mu\nu} \\
 & \quad \phi = \phi_0 \\
 & \quad \tilde{\phi}^i = \tilde{\phi}_0^i
 \end{aligned}
 \tag{26}$$

$$\begin{aligned}
 & = \iint \left\{ \phi_0^2 \frac{\delta^2 A}{\delta \phi^2} + \sum_i (\tilde{\phi}_0^i)^2 \frac{\delta^2 A}{(\delta \tilde{\phi}^i)^2} \right\} \Bigg| , \\
 & \quad g_{\mu\nu} = \eta_{\mu\nu} \\
 & \quad \phi = \phi_0 \\
 & \quad \tilde{\phi}^i = \tilde{\phi}_0^i
 \end{aligned}$$

The terms $\sim \partial^2$ determine Newton's constant:

$$\frac{1}{16\pi G} = + \frac{1}{12} \left(-Z\phi_0^2 - \sum_i (\tilde{\phi}_0^i)^2 \right) . \tag{27}$$

There is no difficulty in satisfying $G > 0$, if we take ϕ to be a ghost field as before, $Z < 0$, with $\phi_0 \sim$ Planck mass, while all other scale fields have expectation values far below that, $\tilde{\phi}_0^i \ll \phi_0$.

However, eq. (26) must also be true for the terms at zero momentum:

$$Zm^2\phi_0^2 + \sum_i \tilde{m}_i^2 (\tilde{\phi}_0^i)^2 = 0 . \tag{28}$$

Likewise, the generalization of eqs. (11) and (12) give

$$Zg_3\phi_0^3 + \sum_{i,j,k} \tilde{g}_3^{ijk} \tilde{\phi}_0^i \tilde{\phi}_0^j \tilde{\phi}_0^k = 0 , \tag{29}$$

while that of eqs. (14) and (15) yields

$$Z g_4 \phi_0^4 + \sum_{i,j,k,l} \tilde{g}_4^{ijkl} \tilde{\phi}_0^i \tilde{\phi}_0^j \tilde{\phi}_0^k \tilde{\phi}_0^l = 0 \quad . \quad (30)$$

Previously, we found that the couplings of the scalar ϕ had to vanish at zero momentum, like those of a dilaton. When there are several scalar fields, we see that only sums over all scalar fields with $\phi_0 \neq 0$ need vanish, and there is no dilaton.

Algebraically, it is not difficult to satisfy these relations. Since by assumption $Z < 0$, all other couplings and masses can be taken to be positive.

The problem is that the resulting theory does not seem to make sense. Consider just eq. (28). There certainly has to be at least one field below the Planck scale which acquires a vacuum expectation value, $\tilde{\phi}_0^i \neq 0$, with non-zero mass, $\tilde{m}_i^2 \neq 0$. If nothing else, in QCD, $\langle \bar{\psi}\psi \rangle \neq 0$, and there is no 0^+ meson lighter than the pion. Hence $m^2 \neq 0$. But if $m^2 \neq 0$, the ϕ -dependent part of the effective action is no longer conformally invariant, and so the transformation of eq. (22) doesn't serve to eliminate ϕ_q : the mass term becomes

$$\int d^4x \sqrt{g} m^2 \phi_q^2$$

$$\xrightarrow{g_{\mu\nu} \rightarrow g'_{\mu\nu}} \int d^4x \sqrt{g'} \frac{m^2}{(1+\phi_q/\phi_0)^4} \phi_q^2 \quad . \quad (31)$$

In fact, one can convince oneself that there is no local redefinition of the fields which will serve to eliminate ϕ_q from the effective action.

Perhaps the same miracle which serves to make the higher derivative part of the gravitational action unitary could also work for the ϕ_q field. However, the problem with higher derivative gravity concerns ghost states with masses at the Planck scale.^{5,6} From eq. (27), $m^2 \sim \tilde{m}_1^2 (\tilde{\phi}_0^1 / \phi_0)^2$, so if $\tilde{\phi}_0^1 \ll \phi_0$, the ϕ_q mass is much smaller than that of the other physical fields, $m \ll m_1$. It seems dubious that a mechanism to implement unitarity at the Planck scale would care about, or affect, an extremely light field. Even so, since the unitarity of higher derivative gravity remains in doubt, logically this possibility cannot be excluded.

A final hope is that the ϕ_q ghost is "eaten" by some sort of conformal Higgs effect, but I am unable to imagine how this could actually come about.

REFERENCES

1. B. Zumino, Nucl. Phys. B89, 535 (1975).
2. The matter fields are scaled as in eq. (1), with $\lambda(x) = \lambda = \text{constant}$, but instead of scaling $g_{\mu\nu}$, the space-time coordinates do so, $x_\mu \rightarrow e^\lambda x_\mu$. The dilatation current for this symmetry is $\sim x_\mu T_m^{\mu\nu}$, where $T_m^{\mu\nu}$ includes only the matter fields (ref. 3).
3. B. Zumino, in Lectures on Elementary Particles and Quantum Field Theory, Vol. 2, MIT Press (1970).
4. To be precise, gravitational fields may contribute to J_D^μ , and so produce a dilaton, but the coupling of the dilaton to any field will always vanish at zero momentum. This can be understood from an effective Lagrangian approach, ref. 3. For any term which is not scale symmetric, such as $\int d^4x \sqrt{g} R$ or $\int d^4x \sqrt{g} \phi_q^2$, factors of a

dilaton field $\sigma(x)$ are added to ensure scale symmetry. For example, these terms become $\int d^4x \sqrt{g} R \exp(2\sigma(x))$ and $\int d^4x \sqrt{g} \phi_q^2 \exp(2\sigma(x))$. But if powers of $\exp(\sigma(x))$ are added to every term in the effective action with dimensions of mass, by redefining the gravitational and matter fields, all zero-momentum couplings of $\sigma(x)$ can be chosen to vanish. The dilaton will then couple to other fields only through momentum dependent terms, such as its kinetic energy, $\sim \int d^4x \sqrt{g} g^{\mu\nu} (\partial_\mu \sigma)(\partial_\nu \sigma)$. This does not happen for theories which are scale-symmetric in flat, but not curved, space-time. While powers of $\exp(\sigma(x))$ are added to all dimensional terms in the matter part of the action, the gravitational terms, e.g. $\int d^4x \sqrt{g} R$, are unaffected, since the scale symmetry does not involve $g_{\mu\nu}$ (ref. 2). By redefinition of the fields, the dilaton can be removed from the matter action, but the gravitational action is now $\int d^4x \sqrt{g} R \exp(-2\sigma(x))$, etc. This is a Brans-Dicke type of gravity: C.J. Isham, A. Salam, and J. Strathdee, Ann. of Phys. 62, 98 (1971).

5. E.T. Tomboulis, Phys. Rev. Lett. 52, 1173 (1984).
6. I. Antoniadis and N.C. Tsamis, Phys. Lett. 144B, 55 (1984): SLAC-PUB-3297 (March, 1984).
7. M.J. Duff, Nucl. Phys. B125, 334 (1977).
8. E.S. Fradkin and A.A. Tseytlin, Budapest preprint KFKI-1983-116. A. Strominger and V.P. Nair, Phys. Rev. D30, 2528 (1984).
9. F. Englert, C. Truffin, and R. Gastmans, Nucl. Phys. B117, 407 (1976).

10. Contrary to my postulates, the theories of Antoniadis and Tsamis, ref. 6, have non-zero β -functions but no conformal anomaly. While in general the effective potential they compute is an involved function of ϕ/μ , they go on to impose a condition on ϕ_0/μ which ends up setting $V(\phi) = 0$.
11. S.Deser, Ann. of Phys. 59, 248 (1970).